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Students' Mathematics Comprehension and Previous Mathematics Performance (PMP): Its Impact on Students' Conceptual Understanding in Determining Area of Plane Regions in Integral Calculus

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ABSTRACT

The study aimed to determine the levels of students' mathematics comprehension and previous mathematics performance and their conceptual understanding in finding the area of plane regions in Integral calculus. It also determined the influence of students' personal attributes on their conceptual understanding of finding the area in calculus. A 7-item teacher-made open-ended test was administered after the discussion of finding the area of plane regions between curves to measure students' conceptual understanding of the topic and questionnaires to determine the personal attributes. The data collected were analyzed using mean, percentage, standard deviation and multiple linear regressions. Results revealed that students' previous mathematics background is an important factor to students' to possess a profound understanding of the concept of finding the area of plane regions in Integral calculus; while mathematics comprehension does not hinder students' conceptual understanding in determining the area of plane regions in Integral Calculus. The researcher then recommends that teachers should teach mathematics subject with more emphasis on the conceptual understanding rather than merely stressing the procedures, students should have mastered the foundation subjects to excel in Calculus like Arithmetic and Algebra; and the mathematics faculty of MUST should conduct training to high school mathematics teachers on how to develop the conceptual understanding of mathematics.

Keywords: mathematics comprehension, previous mathematics performance, conceptual understanding, area of plane regions, integral calculus

INTRODUCTION

Mathematics is an integral component of an educative process. It had been included in the educational curriculum to foster mental discipline and development to make the whole person intellectually competent with logical reasoning, spatial visualization, analysis and abstract thought, (Ministry of Education, Singapore 2006). In the highly competitive and technological world, there is a pressing need for every student to be equipped with the analytical skills to be competent and efficient members of the society. Many people even consider mathematics as a key to open future career options.

Calculus as a branch of mathematics is important to many disciplines. It has been the forefront of educational research during the last two decades. It serves as an excellent tool for solving a variety of problems. Calculus is the key to understanding the systems of changes in society, biological, and the physical sciences (Douglas, 1986). The majority of college students in mathematics, science and engineering, believed that the Calculus supplies a strong foundation in their analytical and computational skills which are deemed essential in the higher level mathematics. This challenging course requires each student to possess well-developed critical thinking skills and excellent background of its pre-requisite subjects. However, despite being an important core course, students too often do not see its relevance. It can be observed that some students consider calculus as a roadblock rather than a gateway in choosing their careers in life (Pinzka, 1999). As a result, the passing rate for calculus is relatively minimal in most cases most especially in the experience of the researcher.

According to the National Research Council (NRC) of the United States of America, there are five strands in building mathematical proficiency for all students. These are conceptual understanding, procedural understanding, strategic competence, adaptive reasoning, and productive disposition. The present study focuses on the first strand, the conceptual understanding otherwise known as conceptual knowledge, knowledge that is rich in showing relationships (Star, 2005). This strand involves understanding of concepts and recognizing their applications in various situations. The constructs of conceptual understanding are very necessary for every student who will be used later in his career. The first strand is the primary concern of educators because it will help students' ability to analyze that will make them flexible to solve real life problems.

Harnessing and developing conceptual understanding seems unachievable because of students' difficulty and misconceptions on many topics were attributed to several factors. Some of these factors are their weakness to perform algorithms and poor background on the fundamental concepts in algebra which are the necessary background to solve calculus problems. In mathematics classroom, students' have developed conceptual understanding if they have clarity of their thought which are logical and can apply the concepts learned appropriately (Wiggins, 2014).

Area of plane regions is a critical topic in calculus. Students find difficulty or if not they have developed a conceptual misunderstanding on finding areas of plane regions. This problem was attributed to any of the factors mentioned. In fact, in the study of Orton (1983) on integration shows that students are "able to apply, with some facility, the basic techniques of integration . . . [but] further probing indicates that they possess fundamental misunderstanding about the underlying concepts. Orton's results indicate that the procedure of breaking up an area or volume, making use of a limit process, and providing the reasons why such a method works were not part of the students understanding of the integral (Orton, 1983).

Given the above premise, the researcher found interest to investigate the effects of students' mathematics comprehension and the previous mathematics performance (PMP) on the conceptual understanding of finding the area of plane regions.

FRAMEWORK

The concept is a mental construct; general notion or idea formed from particular examples, situation or experiences. It includes both tangible and abstract ideas overarching understanding which was derived from examples (Wiggins and McTighe, 2006). On the other hand, understanding is the ability of a person to perform fluently in a given task or situation.

Students enter college with many experiences that are bound to shape their learning about new mathematical concepts (Tall and Vinner, 1981). Concept learning focuses on the aspect of category formation and the use of concepts to interpret experiences and solve problems (Ormrod, 2003). Information processing theory embraces the ideas of conceptual understanding because it is the essence of meaningful learning and is the key to higher level thinking. In education, proponents of constructivism weigh heavily on the need for conceptual learning since it is the basis of all knowledge constructed by the learner (Mestre, 2002).

The vast literature on constructivism has provided mathematics educators with useful ways to understand learning and learners.

The ability to form concepts allows an individual to make sense of the vast amount of information processed every day. Students were exposed to situations to develop a conceptual understanding of events or processes to become proficient in problem solving, abstract reasoning exercises and generalizing ideas to apply to new situations and make connections to related information (Tall and Vinner, 1981). The training that students had experienced in high school can also influence how they learn mathematics since different schools have different curricula; different teaching styles and students have different learning styles. One function of secondary schools is the preparation of the student to tackle higher education (Lavitad, 2005). Students' experiences make them ready to tackle bigger challenges, and readiness is an important factor to face the challenge.

Comprehension is an important ingredient for learning. It helps the students to understand what they are reading. It allows the reader to interact with the text in a meaningful way, and it is an avenue for deeper learning and understanding of concepts (Smith, 1979).

OBJECTIVES OF THE STUDY

This research aimed to: 1) determine the level of students' mathematics comprehension and previous mathematics performance; 2) determine the level of students' conceptual understanding of finding the area of plane regions in Integral Calculus; and 3) determine the impact of students' mathematics comprehension and previous mathematics performance on their conceptual understanding of finding the area of plane regions in Integral Calculus.

METHODOLOGY

Design and Data Gathering Procedure

The study employed a mixed-method research design specifically the convergent parallel design, (Johnson and Onwegbuzie 2004) described the method as the kind of research in which combined quantitative and qualitative research techniques. Data was collected through survey questionnaires and a test on conceptual understanding on finding areas of plane regions. At the start of classes, students were given a questionnaire to measure their Previous Mathematics Performance (PMP), and Mathematics Comprehension. The

PMP test composed of the 25-item validated test. This test was tried out to the students enrolled in the Integral Calculus in a class of summer 2012 to determine the reliability coefficient. The obtained reliability coefficient of this questionnaire was 0.71. The topics included in the PMP test were taken from the subjects Arithmetic, Algebra, Analytic Geometry, Differential Calculus and basic integration techniques. The same procedure was made to the mathematics comprehension test and after the analysis; the coefficient of reliability computed is 0.68.

After the topic on the area of plane regions has been discussed, a second instrument was given to measure their understanding of the area concept as a geometric application of the definite integral. There were seven (7) open-ended items on finding the area of plane regions. The interview was also conducted to measure further their conceptual understanding. A scoring key or rubrics was utilized in checking their answers.

Respondents of the Study

The respondents of this study were students enrolled in Integral Calculus during the first semester of school year 2012 - 2013 at Mindanao University of Science and Technology (MUST). The respondents were composed of the second year and third-year students coming from the College of Policy Studies and Education Management (CPSEM) and College of Industrial and Information Technology (CIIT). A total of 46 students are respondents of this study. These respondents were handled by the researcher in one class setting and enrolled in Integral Calculus, not in their regular load. Some of these students repeated Integral Calculus and some repeated in their previous mathematics classes. This group was chosen as respondents of the study since the class is heterogeneous.

Statistical Methods

In order to determine the profile of the respondents and their performance in the conceptual understanding test, frequency, percentage, and mean were used. To determine the significant difference in the performance of the respondents regarding their mathematics comprehension and previous mathematics performance, t-test for independent samples was used. Multiple regression analysis was utilized to determine which of the personal attributes have influenced the respondents to score in the conceptual understanding test.

RESULTS AND DISCUSSION

Table 1. Mean Score of the Respondents in the Conceptual Understanding Test Grouped According to the Type of High School Attended and Type of Student

Type of High School	Frequency	Percentage	Mean Score in Conceptual Understanding Test	Descriptive Rating
Public	32	70	30.14	Fair
Private	14	30	47.98	Satisfactory
Type of Student				
Repeater	8	17	37.90	Satisfactory
Non-repeater	38	83	40.22	Satisfactory
Total	**46**	**100**	**39.06**	**Satisfactory**

Table 1 shows that from among the 46 students, only 17% are repeaters and 83% are non-repeaters. It can be noted that majority are taking the subject for the first time in Integral Calculus but, after the interview they revealed that from among the 38 non-repeaters of the subject, some of them have repeated in their previous mathematics subjects, and they took summer class in their Differential Calculus subject, a pre-requisite of the subject Integral Calculus. Also, from the repeaters group, when asked what has been the reason they failed in their previous Integral Calculus class, almost all have reasoned that the subject is really difficult for them and accepted that they need to repeat the subject. Regarding their conceptual understanding test performance when grouped according to the type of student, it can be seen that both groups have a satisfactory performance as indicated by their mean score of 37.90 and 40.22 respectively. A recorded interview of the repeater respondent was shown below:

Researcher: *"Ngano nabagsak man ka sa imong previous na Integral?"*
[Why did you fail in your previous Integral Calculus class?]

Student: *"Lisod man jud kaau sir ui" Unya paspas pajud among maestro, mao pa na dli kaau mi mga kabalo, samot dli jud mi kaapas sa among lesson.Labaw najud sa applications, dli mi kaanswer."*
[The subject is really difficult and the instructor is fast in discussing the lessons especially the applications part, we cannot answer.]

Researcher: *"Unsa man inyong gabuhaton para makasabot mo?"*
[What did you do to understand your lessons?]

Student: *"Na tinabanga nalang mi sir, pero gaka-bagsak japun mi kay dli pud kabalo akong uban classmate."*

[We helped each other but some of my classmates also do not know how to answer so we fail.]

The interview above only shows that aside from the subject being difficult, the teacher also has an influence on the students' failure on the subject. It is also noted that they find difficult dealing with applications, which was due to their weak background in their previous mathematics subject and for their conceptual understanding of mathematics.

On the other hand, out of the 46 respondents, 70% finished their high school from the public school, while 30% are from the private school. It can be noted that most of the respondents are coming from the public secondary schools. They might have studied here at MUST because it is a state university and the tuition is not as high as the private schools. In terms of their conceptual understanding test performance, it can be observed from the table that students coming from the private schools have a higher mean score of 47.98 compared to those students coming from the public schools who only got a mean score of 30.14 which has a descriptive rating of fair which means that they were still in the developing level.

Table 2. Students' Level of Mathematics Comprehension Skills and Previous Mathematics Performance (PMP)

Variables	Mean	Frequency	Percentage
Mathematics Comprehension			
Below Average	3.39	13	28
Average	5.87	30	65
Above Average	8.00	3	7
Previous Mathematics Performance (PMP)			
Below Average	**	0	0
Average	14.8	41	89
Above Average	20.4	5	11
	TOTAL	**46**	**100**

The result of the analysis of the mathematics comprehension skills of the respondents shows that among the 46 respondents, 28% of them were below average (mean score is 1-4) mathematics comprehension skill. This is also indicated by their mean score of 3.39. Out of the 46 respondents, 65% are average (mean score is 5-7) in their mathematics comprehension skill, and they got a mean of 5.87. Only 7% of the respondents showed above average (mean score is 8-10) mathematics comprehension skill. All from the above average group of respondents got a score of eight (8) out of ten. Mathematics comprehension is one important factor to consider in doing mathematics. Students who have a below average comprehension skills in mathematics could not understand the lessons well especially in problem-solving.

Concerning the performance of the students in their previous mathematics, the above table showed that none of these respondents have a below average performance. Because these students, if they are not repeaters, may have mastered already the basic concepts needed since they have repeated in their previous mathematics subjects. The majority of the respondents or 89% of them have an average performance in their previous mathematics topics. This performance was indicated by their mean score of 14.8. Only 11% have an above average performance, with a mean of 20.4. The students' previous mathematics performance was indeed related to the present performance of the students taking up Integral Calculus.

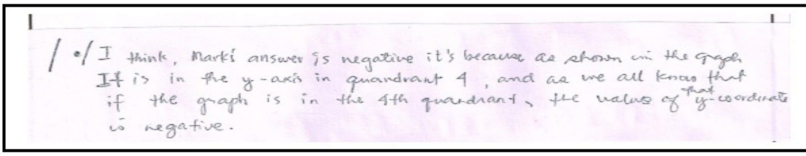
Table 3. Students' Overall Performance in the Conceptual Understanding Test Items

Item	Mean Score	Description
1	2.65	Satisfactory
2	3.07	Satisfactory
3	2.80	Satisfactory
4	3.15	Satisfactory
5	2.20	Fair
6	2.90	Satisfactory
7	2.60	Satisfactory

Legend: *Mean Score* / *Description*

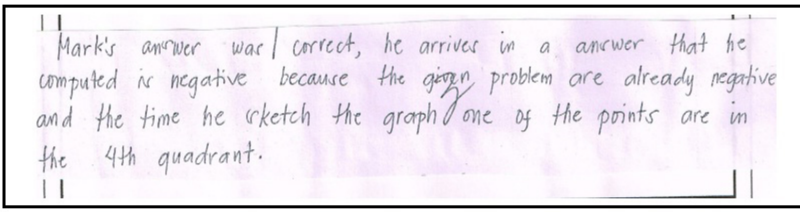
Mean Score	Description
4.5 – 5	Excellent
3.5 – 4.49	Very Satisfactory
2.5 – 3.49	Satisfactory
1.5 – 2.49	Fair
1 – 1.49	Poor

Table 3 above shows the result of the students' performance in the conceptual understanding test items. It can be seen that among the seven problems, students have a fair performance in problem number 5. In problem number 5, students' were asked to explain why the student arrived with a negative area when the region is below the x-axis. The majority of the respondents' reasoned that the area is negative because the region is below the x-axis but failed to establish the idea that area should have always been positive. Some did mention that after getting the area, absolute value will be applied to have a positive area. Sample of students' answer was displayed below:



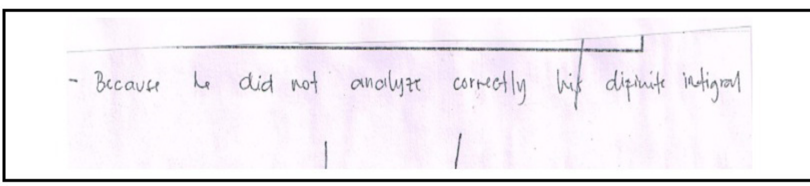
I think, Mark's answer is negative it's because as shown in the graph It is in the y-axis in quadrant 4, and as we all know that if the graph is in the 4th quadrant, the value of that y-coordinate is negative.

Student 1



Mark's answer was correct, he arrives in a answer that he computed is negative because the given problem are already negative and the time he sketch the graph one of the points are in the 4th quadrant.

Student 2



- Because he did not analyze correctly his difinite intigral

Student 3

Figure 1. Sample Output of Three Students for Problem # 5.

Furthermore, it can be noted in Table 3 that students have a satisfactory performance with problem 4. In problem 4, students were shown two responses of students about finding the area enclosed by a parabola. John and David as students were asked to illustrate the area and represent the area by a definite integral. Both students have the correct illustration, John using the vertical rectangular element while David uses the horizontal rectangular strip. When they represent the area of the region, John made a correct definite integral while David, although he has the correct illustration, failed to have a correct definite integral. Respondents were asked to which answer is correct, the majority of the respondents have a correct choice that is John. This situation implies that students have a strong background when it comes to graphing the region but are weak when asked to interpret the area using the definite integral. A sample of one of the respondents' answer was displayed in the next figure.

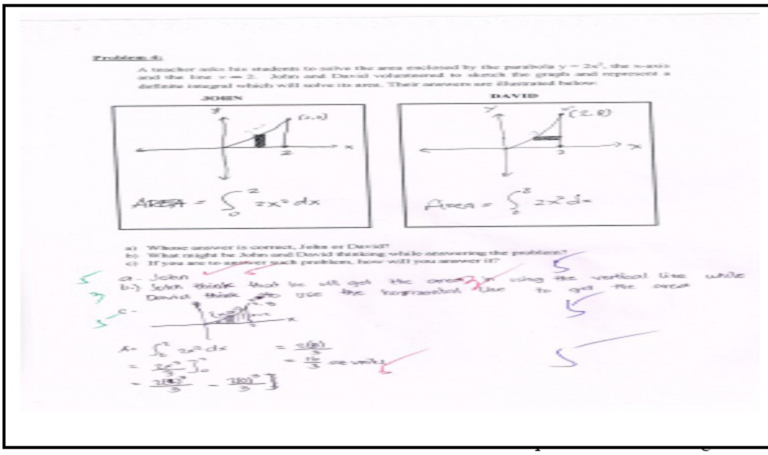


Figure 2. Sample Output of a Student for Problem # 4.

Table 4. Students' Overall Performance in the
Conceptual Understanding Test

Score	Frequency	Percentage	Description
63 – 70	0	0	Excellent
49 – 62	7	15	Very Satisfactory
35 – 48	24	52	Satisfactory
21 – 34	15	33	Fair
Below 21	0	0	Poor
Total	**46**	**100**	

Table 4 above shows the result of the overall performance of students in the conceptual understanding test. The score range was patterned on the rubric standard used in the K-12 curriculum. It can be seen in the table that nobody from the group have an excellent performance as well as students with a poor rating in the test. It can be observed that almost all students have an average performance; seven (15%) have only very satisfactory performance, only fifteen or (33%) have a fair performance and twenty or 52% have a satisfactory performance. This result would mean that students' are doing good in the conceptual understanding test which might be due to their average background in the foundation subjects which is the result of being a repeater of the subject or may be repeating in their previous mathematics subjects. However, still teachers do not settle for less; it should stress the importance of conceptual understanding in all their lessons to increase students' achievement.

Table 5. Regression Analysis of Students' Conceptual Understanding Test Scores with the Selected Variables

Variables	Coefficient	Standard Deviation	t-value	Probability
Mathematics Comprehension	-0.126	0.89	-0.14	0.888
PMP Test	0.942	0.46	2.04	0.048*
Constant	21.65	12.41	1.75	0.089
Standard Error of Estimate = 8.730	R-squared = 14.8	Multiple R = 13.3		

*p<0.05

Table 5 shows the regression analysis used in determining whether mathematics comprehension and previous mathematics performance (PMP) have significantly influenced the students' conceptual understanding test score in determining the area of plane regions. It can be noted that the only factor that gave a significant influence on students' conceptual understanding is the students' previous mathematics performance as indicated by the probability value of 0.048. This result implies that students' strong previous mathematics background is very important to succeed in advanced topics in Calculus. This scenario might be due to their exposure just before they took Integral Calculus and the concepts are still fresh to them. It is true that strong foundation is necessary for upper-level mathematics, but if there is a considerable time gap, concepts may be forgotten. In this group of students, the previous mathematics lesson was just taken the semester immediately, they took Integral Calculus.

The PMP test has a positive influence on the conceptual understanding test score as indicated by the beta-coefficient value of 0.997. This result would mean that to have a profound understanding of finding the area of plane regions in Integral Calculus, a student should be armed with a good background in the foundation topics and students should understand the problem well. The students' mathematics comprehension showed weak influence on the conceptual understanding test scores as indicated by probability values greater than 0.05.

CONCLUSIONS

In light of the above results, the following conclusions were drawn: students' previous mathematics background is a relevant factor for them to have a profound understanding of the concept of finding the area of plane regions in Integral Calculus and students' mathematics comprehension is not a predictor of conceptual understanding in determining the area of plane sections in Integral Calculus.

RECOMMENDATIONS

Based on the findings and conclusion of the study, the following recommendations are forwarded:

1. Calculus teachers can study how conceptual understanding be developed in their teaching of the area of plane regions as an application of the definite integral;
2. Students should have mastered the foundation subjects to excel in Calculus like Arithmetic and Algebra;
3. The mathematics faculty of MUST should conduct training to calculus teachers on how to develop the conceptual understanding of mathematics; and
4. This study can be replicated in a bigger population in different courses to consider other variables that may affect students' conceptual understanding in Calculus and with other mathematics subjects.

LITERATURE CITED

Douglas, R.G.
1986 "The Importance of Calculus in Core Mathematics", Journal of College Science Teaching, Volume 15, pp. 250-251.

Johnson, R.B. & Onwegbuzie, A.J.
2004 "Mixed Methods Research: A Research Paradigm Whose Time has Come", Educational Researcher, Vol. 33(7), pp. 14-26.

Lavitad, N. D.
2005 Mathematical Readiness and Misconceptions of Freshman Engineering Students in Cagayan de Oro College during the First Semester of SY 2004-2005. Mindanao University of Science and Technology, Cagayan de Oro City.

Mestre, J.

2002 Transfer of Learning: Issues and Research Agenda (No. NSF03-212). Arlington, VA: National Science Foundation.

Ministry of Education, Singapore

2006

Ormrod, J.E.

2003 Human Learning, Columbus,OH, Prentice Hall.

Orton, A.

1983 "Students' Understanding of Integration". Educational Studies in Mathematics, V14, pp1-18.

Pinzka, M.K.

1999 "The Relationship between College Calculus Students' Understanding of Function and their Understanding of Derivative", University of Minnesota, USA.

Smith, F.

1979 Reading without Nonsense". New York, Teachers College Press. National Research Council (NRC), United States of America (USA), 2001.

Star, J.R.

2005 "Reconceptualizing Procedural Knowledge", Journal for Research. In Mathematics Education, Volume 36, pp. 404-411.

Tall, D.

1996 Functions and Calculus. In International Handbook of Mathematics Education. Part 1. Eds.Bishop, A., Clements, K., Keitel, C., Kilpatrick, J., Laborde, C. – Dordrecht, The Netherlands: Kluwer. Pp. 289-325.

Tall, D.O. & Vinner, S.

1981 Concept Image and Concept Definition in Mathematics with Special Reference to Limits and Continuity, Educational Studies in Mathematics, 12, 151-169.

Wiggins, G.
2014 Conceptual Understanding in Mathematics, https://grantwiggins.wordpress.com/2014/04/23/conceptual-understanding-in-mathematics/

Wiggins & McTighe
2006 Understanding by Design, Pearson: Merrill Prentice Hall, p.24. ISBN 0-13-195084-3.