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The Effects of Problem Posing and Sense-Making on Students' Conceptual Understanding and Anxiety towards Mathematics

ROSIE G. TAN
ORCID NO. 0000-0002-4424-8193
rosiegtan@gmail.com

CHARITA A. LUNA
ORCID NO. 0000-0002-9026-2121
charita_luna@yahoo.com

RHODA A. NAMOCO
ORCID NO. 0000-0003-0626-1431
rhodaagdeppanamoco@gmail.com

University of Science and Technology of Southern Philippines
Cagayan de Oro City, Philippines

ABSTRACT

This study examined the effects of problem posing and sense-making on students' conceptual understanding and mathematics anxiety. It utilized a pretest-posttest quasi-experimental non-equivalent control group design to gather the data. The researcher made use of the teacher made test and mathematics anxiety test with a reliability coefficient of 0.89 and 0.88, respectively. Two intact classes of Grade 8 students at Bugo National High School, Bugo Cagayan de Oro City were randomly assigned as the control group and the other as the experimental group. The analysis of covariance (ANCOVA) revealed that the conceptual understanding of students taught using problem posing and sense-making has significantly higher than those taught using a conventional method of teaching. However, the mathematics anxiety of the students exposed to the two methods of teaching was comparable.

Keywords: Problem posing, sense-making, conceptual understanding, mathematics anxiety

INTRODUCTION

Conceptual understanding is a hot topic in the teaching and learning of mathematics. Mathematics learned conceptually enables students to use their knowledge flexibly, and empower them to become more proficient at problem-solving (NCMT, 2000). For students to understand mathematics conceptually, these researchers believe that they must have the opportunity to explore the mathematical concepts by themselves, connect it to their previous knowledge and then use various strategies to complete the task. It means that the teaching-learning process must shift from engaging students in a procedural activity to more complex tasks. It has been observed that most mathematics teachers teach mathematics with more practice. They show each step and then ask the students to answer similar examples following the same procedure. In most cases, students are not given the opportunity to explore mathematical concepts by themselves.

Correct procedures in answering a mathematical task are important, but the knowledge of when, why and how to use a particular procedure receives increasing attention. This suggests the need to emphasize sense-making in mathematics classrooms. According to Van den Kieboom and Margiera (2012), mathematical sense-making is one's ability to critically analyze concepts, situations, and contexts and draw connections with other knowledge.

Practice-solving problems can support the improvement of conceptual knowledge when constructed appropriately (McNeil, Chesney, Matthews, Fyfe, Petersen, L. A., Dunwiddie, A. E., & Wheeler, M. C., 2012). However, reforms place problem posing with equal emphasis on problem-solving. In the Professional Teaching Standards, it proposes that students should be given the opportunity to formulate problems from given situations and create new problems by modifying the conditions of a given problem (El Sayed, 2002). Brown and Walter (2005) added that problem posing is more important than solving problems because this activity has a rich application. It can assess students' conception of numbers and operation (Rathouz, 2001). According to Bogomolny (2015), inventing problems is a great tool for learning the current course and mathematics in general.

Moreover, problem posing is impossible without sense-making. In posing a problem, one needs to make sense among mathematical concepts, quantities, arithmetic properties, and the procedure of solving the problem. To this end, these researchers believed that sense- making is inseparable with problem posing. Combining these two methods of teaching push students to think deeply. Problem posing and sense-making activities require students to be well versed in all mathematical concepts and how to apply these across a wide range of problem context. In addition, this method of teaching is new and complex, it might affect students' anxiety towards mathematics.

With these end in view, this study attempted to investigate the effect of problem posing and sense-making on students' conceptual understanding and mathematics anxiety.

FRAMEWORK

This research is anchored on the theories of constructivism. It embraces that learning is an active process where the students are involved in the construction of knowledge. Examples of these theories are the discovery learning theory of Bruner (1961), assimilation and accommodation theory of Piaget (1958), meaningful learning of Ausubel's (1968) and Zone of Proximal Development (ZPD) of Vygotsky (1978).

The discovery learning theory of Bruner (1961) highlighted that facts, relationship, and concepts are best learned through discovery and inquiry. To Bruner, the ability to invent for oneself is the most important outcome of learning (Mc Leod, 2008). Bruner assumed that intrinsic motivation can be developed by discovering for oneself and that what is learned will be more easily remembered. Such a theory is related to this current study because as students pose a problem, they think of facts, relationships, and concepts so that their constructed problem makes sense. They discover the connections between facts as they solve the problem they posed. Thus, they have a chance to develop conceptual understanding. In addition, when students interact with the problems, struggle to answer the questions, make sense of the concepts used in their algorithm, they are more likely to remember concepts and facts.

Piaget (1958) states that assimilation is a process of comparing new information into already learned knowledge while accommodation is the process of adjusting the information in order to interact with the existing knowledge. These processes allow the students to differentiate the current information that they have, and then add the new information acquired. Piaget held that problem-solving skills cannot be taught, they must be discovered. He added that the more active the students are, the more likely they remember concepts. This theory is related TO this study because, in this study, the students are actively involved; they pose their own problem, solve the posed problems, and elucidate their process as well. When their posed problem does not make sense, they revised it. The students were asked to reconsider what was once misunderstood and reframe new experiences. The students develop new outlooks and improve their understanding.

The meaningful learning theory of Ausubel's (1968) states that learning of new information relies on what is previously known. Ausubel supposed that learning is an active process, the learners look for new information and integrate this with their existing knowledge. This theory asserts that facts, ideas and thoughts learned meaningfully are retained longer. To this end, teachers need to offer activities which engage the mind which is a feature of problem posing and sense-making method. In this study, the students are encouraged to think and explain their reasoning instead of memorizing and reciting facts. This progression is necessary to strengthen the connection between mathematical concepts already learned and to develop understanding of the new concepts and retain more ideas.

Learning is more meaningful when students are working with their peers. Hence, this study is also anchored on the Zone of Proximal Development (ZPD) by Vygotsky (1978). The ZPD asserts that working with peers reduced students' anxiety and improve learning.

Grounded on the forgoing theories, the present study focused on the examination of the effect of problem posing and sense-making on students' conceptual understanding, and mathematics anxiety.

OBJECTIVES OF THE STUDY

This study explored the effects of problem posing and sense-making on students' conceptual understanding and mathematics anxiety as compared with the conventional method of teaching. It aimed to: a) determine the level of grade 8 students' conceptual understanding of special products, factoring, and rational algebraic expression; b) compare the conceptual understanding of students on special products factoring, and rational algebraic expression as influenced by the methods of teaching; c) determine the level of students' mathematics anxiety; and d) compare the mathematics anxiety of students as influenced by the methods of teaching.

METHODS

This investigation utilized a pretest-posttest quasi-experimental non-equivalent control group design. The researchers randomly chose two intact classes from among 5 sections of grade 8 level of Bugo National High

School. One section was randomly assigned as the experimental group and the other as the control group. The experimental group was taught using problem posing and sense-making as a method of teaching while the experimental group was taught using the conventional method of teaching using the K-12 material as a reference.

First, the participants answered the teacher-made conceptual understanding test about special products, factoring, and rational algebraic expressions. This test is 26 points. It is composed of 10 items multiple choice and 4 open-ended questions. The multiple-choice items were scored using two scales, 0 or 1. 0 if the response was wrong and 1 if it was correct. The open-ended questions were scored using four-point scale; 0 if no work was shown, 1 if some parts of the solution were correctly done but with no evidence of connection, 2 if half of the solution was correctly done with some evident conceptual errors, 3 if the solution was correct but not complete with some minor error in the notation or in the computation, 4 if the solution was complete and correct. This test has a reliability coefficient of 0.89.

The items in this instrument were designed to measure students' conceptual understanding, which was constructed based on the study of Teachey (2003). These items required students' flexibility, reversibility, generalization and transfer skills. The total score was considered as the students' achievement on the conceptual understanding test. The criteria for the students' level of conceptual understanding were based on the National Education Testing and Research Center (NETRC), Department of Education (DepEd). A score of 75% and above means that the student has a mastery of the concepts; 50% to less than 75%, near mastery; and below 50% means low mastery of concepts (Sarmiento,2006).

On the second day, the students answered the mathematics anxiety test for 15 minutes. It composed of 10 statements with a 5-point Likert scale such as strongly agree, agree, uncertain, disagree and strongly disagree. This test has a reliability coefficient of 0.88. Right after this test, instruction began.

In the experimental group, the teacher posed a mathematical problem with answers given, but she did not explain how the problem is solved. Instead, she asked the students to pose and solve their own problem. This problem must be related to students' prior knowledge so that every student can relate the activity. After the given time allotment, the students were asked to exchange their work with their classmate. They were asked to explain each other's work in a small group or with their partner. Then, the teacher added another condition to her problem and required the students to do the same. This time, the students were given the opportunity to explore the answer to their problem. After which, the teacher gave a short lecture to deepen the students' understanding and required the students to solved their problem.

Occasionally, the teacher selected two erroneous problems created by the students and presented these in class. The students were challenged to find the error in the problems to excite enthusiasm to make meaning and usefulness of their prior knowledge. They were encouraged to present the error found and were required to explain why the given problem was wrong. They were also challenged to revise the problem to make it correct and show the solution of the revised problem on the board.

During the assessment, students were requested to pose a problem and let their classmate solve it. They were instructed to exchange problems in a clockwise direction. After the count of two, they solve the problem received in 5 minutes. The process continued until 5 problems were solved by each student. This manner of conducting the class was done during the duration of the experiment.

As to the control group it likewise followed the K-12 guide which included interaction. The class started with a drill of three exercises, the students' work were checked to assess if the students were ready to proceed to the new activity. After the exercises, the teacher proceeded to the activity in the K-12 guide, asking the students to work on the activity. The activities were done either with the whole class, with a small group or in pairs. There were also questions that followed after the activity, but the questions did not require strict sense-making. It was followed by a lecture then another activity, exercises, and more seat work. The same procedure was done in all lessons in the K-12 guide.

After the specified topics of the study were covered, the posttest was administered with the same instrument used in the pretest.

RESULTS & DISCUSSION

The results of the analysis of the data collected are shown in the following tables:

Table 1

Students' Scores on Conceptual Understanding Test

Indicators of Conceptual Understanding	Experimental Group				Control Group			
	Pretest		Posttest		Pretest		Posttest	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Flexibility	0.25	0.43	3.12	1.85	0.24	0.49	2.61	1.71
Reversibility	0.22	0.41	2.96	2.13	0.27	0.45	2.13	1.55
Generalization	0.29	0.55	1.46	0.55	0.32	0.52	1.12	0.64
Transfer	0.61	0.76	7.13	3.15	0.73	0.90	3.22	1.86
Overall Score	1.37	1.18	14.67	6.53	1.56	1.07	9.08	4.58
Level of Conceptual Understanding	Low		Near Mastery		Low		Low	

Table 1 shows the mean and standard deviation of the indicators of conceptual understanding and the level of students' conceptual understanding of special products, factoring, and rational algebraic expression. Note that in the pretest the students have low scores in every indicator of conceptual understanding. For the questions requiring flexibility, the experimental group got a mean of 0.25 out of 6 while the control group got 0.24. For the questions requiring reversibility, the experimental group got a mean of 0.22 out of 6 while the control group got 0.27. For the questions requiring generalization, the experimental group got a mean of 0.29 out of 2 while the control group got 0.32. For the questions requiring transfer skills, the experimental group got a mean of 0.61 out of 12 while the control group got 0.74. These scores indicate that the students in both groups do not possess flexibility, reversibility and transfer capability before the treatment. Generally, both experimental and control groups got a low conceptual understanding with an overall mean of 1.37 and 1.56, respectively. This further indicates that the students do not have a background on the topics prior to the experiment.

The table also shows that the standard deviations in the pretest are all small which means that their scores are homogeneous. Specifically, for the questions requiring flexibility, the experimental and control groups got a standard deviation of 0.43 and 0.49, respectively. For the questions requiring reversibility, the experimental group got a standard deviation of 0.41 while the control group got 0.45. For the questions requiring generalization, the standard deviations of the experimental and control groups are 0.55 and 0.52, respectively. Also, for the questions requiring transfer capability, the experimental group got a standard deviation of 0.76 while the control group got 0.90. The overall standard deviation for the students' conceptual understanding is 1.18 and 1.07 by the experimental and control groups, respectively. This means that most of the students in both groups got very low scores in conceptual understanding test before the treatment.

In the posttest, note that students have increased their mean scores. The experimental group got 3.12 for flexibility, 2.96 for reversibility, 1.46 for generalizability, and 7.13 for transfer skill. In the same test, the control group got 2.61 for flexibility, 2.13 for reversibility, 1.12 for generalizability and 3.22 for transfer skill. It can also be noted that for every indicator, the experimental group got higher mean than the control group, especially for questions requiring transfer skills where the experimental group got 3.91 higher than the control group. The overall mean of the experimental group is 14.67 out of 26 while the control group got 9.08. It can be observed that there is a difference of 5.59 in favor of the experimental group. The scores gained by the experimental group after the treatment have increased their conceptual understanding to near mastery level while the control group remained in the low level of conceptual understanding.

The standard deviations in the posttest have increased which means that their scores became heterogeneous, some students got high while others got low scores. However, the spread of the scores in the experimental group is wider than the control group, except for the questions requiring generalization skills. Particularly, for the questions requiring flexibility, the experimental and control groups got a standard deviation of 1.85 and 1.71, respectively. For the questions requiring reversibility, the experimental group got a standard deviation of 2.13 while the control group got 1.55. Also, for the questions requiring transfer capability, the experimental group got a standard deviation of 3.15 while the control group got 1.86. However, for the questions requiring generalization, the standard deviations of the experimental is 0.55, and the control group is 0.64. The overall standard deviation for the experimental is 6.53 while the control group is 4.58. This means that the scores of the experimental group for the conceptual understanding test are having wider dispersion, that is, some got very high while others got very low.

To determine if there is a significant difference between the conceptual understanding of the experimental and control group, the Analysis of Covariance (ANCOVA) was employed.

Table 2

One-way ANCOVA Summary for Students' Conceptual Understanding

Source	DF	Adj SS	AdjMS	F	P
Treatment Effect	2	761.22	380.61	12.53	0.001
Error	78	23.68.4	30.36		
Total	80	3184.1			

**Significant at 0.05 level*

Table 2 presents the summary of the analysis of covariance of pretest and posttest scores for students' conceptual understanding of the experimental and control groups. The analysis yielded a computed F-ratio 12.53 and a probability-value of 0.001 which is less than the 0.05 level of significance. This led to the rejection of the null hypothesis. This means that there is sufficient evidence to conclude that the conceptual understanding of the students exposed to problem posing and sense-making is significantly higher than those exposed to the conventional method of teaching. This is because when students were exposed to problem posing and sense-making they were provided with the opportunity to exercise their mind. This method of teaching requires high cognitive demand task to keep them thinking deeply which led to better and improved conceptual understanding. This result confirms the findings of Stalling (2007) and Mestre (2000) that posing mathematical problems will raise students' level of mental work and demonstrate mathematical understanding. It also agrees with Tobias (2014) and Bruning (2004) who indicated that presenting to students situations which can connect ideas and strategies may support their conceptual understanding. Likewise, the result confirms the findings of Rafols (2003), Polizon (2003), Rathouz (2001) and Knuth, Peterson (2002) that students exposed to problem posing were able to solve word problems and that problem posing is an effective method to enhance students' problem-solving ability. Furthermore, it confirms the theory of Bernardo (2001) that the ability to construct problem increases transfer skills.

Another reason might be due to the students' involvement in the class. Students in this group were directly involved in the learning process. The progression of the lesson depends on the problem they posed. Also, the methods of teaching provoke students' supremacy. They challenged each other by posing a problem and let others solved it. They were asked also asked to criticize the problem posed by their classmates. So every time they posed a problem, they saw to it that their problem made sense so that no error would be found by their classmates.

Table 3

Level of Students' Mathematics Anxiety

	Experimental Group		Control Group	
	Pretest	Posttest	Pretest	Posttest
Overall Mean	2.84	2.76	2.97	2.90
SD	0.56	5.04	0.56	7.63
Descriptive Level	Undecided	Undecided	Undecided	Undecided

Table 3 shows the mean, standard deviation, and level of students' mathematics anxiety. It can be gleaned in the table that in the pretest, the mean scores of both groups are between 2.50 to 3.00, which means that both groups had almost the same feeling towards mathematics before the treatment. That is, they were uncertain of their emotions towards mathematics at the start of the study.

After the treatment, the anxiety of the experimental group decreased by about 0.08, and the control group had also decreased by about 0.07. This may be because the teacher may already establish rapport with her students. The group activity may also help them felt at ease and became more comfortable. In addition, students were allowed to speak in dialect making them easy to express their thought and asked for clarifications if things were not clear to them, thus, their anxiety was reduced.

The standard deviation of the experimental group and control group in the pretest are the same. Both groups got the standard deviation of 0.56 which is very small. This means that the feelings of the participants in the experimental and control groups were almost the same. Most of them were not sure of their emotions towards mathematics. In the posttest, it can be seen from the table that the standard deviation of the experimental and control groups increased. This means that there were changes in the anxiety level of the participants after the treatment. Their feeling towards mathematics became diverse. There were students who became very anxious while others were not anxious at all.

To determine if there is a significant difference in the mathematics anxiety between the experimental and control groups, the ANCOVA was used.

Table 4

One-way ANCOVA Summary for Students' Mathematics Anxiety

Source	DF	Adj SS	AdjMS	F	P
Treatment Effect	1	0.1004	0.1004	0.29	0.589
Error	79	26.9271	0.3408		
Total	81	33.8362			

Not Significant at 0.05 level

Table 4 shows the summary of the analysis of covariance of pretest and posttest scores for students' mathematics anxiety of the experimental and control groups. The analysis yielded a computed F-ratio of 0.29 with a probability-value of 0.589 which is greater than the 0.05 level of significance. This led to the none rejection of the null hypothesis. This means that there is no enough evidence to conclude that the mathematics anxiety of the experimental is better than the control groups. Problem posing and sense-making and conventional method of teaching had both reduced students' anxiety towards mathematics. Although problem posing and sense-making method is new to them, students showed interest and tried their best perform the task. According to Fosnot and Dolk (2001) and Knuth, et al. (2002), problem posing helps students become motivated. Knowlton and Sharp (2003) added that problem-based instruction is highly motivating to students. The amount of mathematics anxiety reduced in the experimental group is not sufficient to come up with a significant difference in the mathematics of students exposed to the conventional method of teaching.

CONCLUSIONS

Based on the findings of the study, it can be inferred that problem posing and sense making are effective in enhancing the conceptual understanding of grade 8 students in Bugo National High School, Bugo, Cagayan de Oro City on special products, factoring, and rational algebraic expression as compared to the conventional method of teaching. This method is comparable with the conventional method in reducing the students' mathematics anxiety.

RECOMMENDATIONS

The researchers recommend the following:

1. Teachers may require students to pose their own problem and let them engage in sense-making tasks to enhance their conceptual understanding;

2. School principals and supervisors are encouraged to support the implementation of problem posing and sense making in mathematics classrooms to cultivate mathematics fluency; and

3. Similar studies may be conducted to a wider scope using a different population in other learning institutions to promote the generalizability of the results.

LITERATURE CITED

Ausubel, D. (1968). *Educational Psychology: A Cognitive View*, New York: Holt, Rinehart & Winston.

Bernardo, A. (2001). Analogical problem construction and transfer in mathematical problem solving. *Educational psychology, 21*(2), 137-150.

Bogomolny,A. (2015). Problem Posing vs Problem Solving *from Interactive Mathematics Miscellany andPuzzles.* http://www.cut-theknot.org/Mset99/posing.shtml.

Brown and Walter (2005). *The art of problem posing (3rd ed.).* Mahwah, NJ: Lawrence Erlbaum Associates.

Bruner, J. (1961). The act of discovery. *Harvard Educational Review, 31*, 21-32.

Bruning, R. (2004). Cognitive approaches to mathematics. In R. Bruning, G. Schraw, M. Norby, & R. Ronning, *Cognitive psychology and instruction*, 4th ed. (Chapter 14, pp. 1 – 28). New York, NY: Merrill Princeton Hall.

El Sayed, R. (2002). Effectiveness of Problem Posing Strategies on Prospective Mathematics Teachers' Problem Solving Performance. *Journal of Science and Mathematics Education in S.E. Asia, 15*(1). http://www.recsam.edu.my/R&D_Journals/YEAR2002/2002Vol25No1/56-69.pdf 56.

Fosnot, K. and Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition and subtraction*. Portsmouth NH: Heinemann.

Knowlton & Sharp (2003). *The effectiveness of problem-based learning in the development of critical thinking. Laxington*, MA: D.C. health and Company.

Knuth, E. and Peterson, B. (2002). Fostering Mathematical Curiosity, *Mathematics Teacher*, 95 (2), 126.

McNeil, N. M., Chesney, D. L., Matthews, P. G., Fyfe, E. R., Petersen, L. A., Dunwiddie, A. E., & Wheeler, M. C. (2012, June 25). It Pays to be Organized: Organizing Arithmetic Practice Around Equivalent Values Facilitates Understanding of Math Equivalence. *Journal of Educational Psychology*. Advance online publication. doi: 10.1037/a0028997.

Mc. Leod, S. (2008). *Bruner*. Available at http://www.Simplypsychology.org

Mestre, J. (2000). *Progress in research: the interplay among theory, research questions and measurement techniques*. In R. Lesh & A. Kelly (Eds)., Handbook of research design in mathematics and science education. Marwah, NJ: Lawrence Erlbaum.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Patton M. (2002). *Qualitaitive Research & Evaluation (3rd Edition)*. Thousand Oaks, CA: Sage Publication, Inc.

Piaget, J. (1958). The growth of logical thinking from childhood to adolescence. *AMC, 10*, 12. Available At: http://www.simplypsychology.org/piaget.html#adaptation.

Polizon, M. (2003). *Problem Posing Approach in Teaching Problem Solving: Its Impact on Students' Achievement in Plane Trigonometry*. (Unpublished thesis). Mindanao Polytechnic State College, Cagayan de oro City.

Rafols, G. (2003). *Problem Posing: Its impact on Problem Solving in Fith Grade Mathematics.* (Unpublished Thesis). Mindanao Polythecnic State College.

Rathouz, M. (2011, March). Making Sense. *Mathematics Teaching in the Middle School*, 16, 430.

Stallings, L. (2007). "See a Different Mathematics," *mathematics teaching in the Middle School* 13 (4): 212-217.

Teachey. A. (2003). *Investigations in conceptual understanding of polynomial functions and the impact of mathematical belief systems on achievement in an accelerated summer program for gifted students.* Dissertation: Thesis (Ph.D.)-North Carolina State University. [Online]. Available at http://www.worldcat.org

Tobias, J. (2014). Mixing Strategies to Compare Fractions. *Mathematics Teaching in the Middle School*, *19*(6). February 2014, pp. 376-381.

Van de Kieboom, L. and Magiera M, (2012). Algebraic Representation. *Mathematics Teaching in the Middle School*. Vol 17. No, 6, 353.

Vygotsky, L. (1978). Interaction between learning and development. From: *Mind and Society* (pp.79-91. Cambridge, MA: Harvard University Press.

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